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Comparison of two process capabilities by using indices C_{pm}: an application to a color STN display

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Abstract *Recent years have seen mounting interest in measuring process performance in the manufacturing industry. Analysis of process capability indices allows a production department to trace and improve a poor process to enhance quality and satisfy customers. Process capability analysis can also serve as an important reference in determining strategies to improve global product quality. Since* C_p *and* C_{pk} *failed to account for process centering, index* C_{pm} *was developed, which considers process centering and is suitable for processes of the nominal-the-best type. Other indices like* C_{pu} *and* C_{pl} *also exist, and are used for unilateral specification. Chou developed a procedure using estimators of* C_p, C_{pu} *and* C_{pl} *to allow practitioners to determine whether two processes are equally capable. For bilateral specification processes, index* C_p *fails to measure process yield and process centering, and thus the index* C_{pm} *is used to develop a similar procedure to help practitioners determine whether or not two processes are equally capable. Naturally, decisions made using the procedure to select the better supplier are more reliable than decisions made using other methods.*

1. Introduction

Process capability indices (PCIs), which aim to provide a numerical measure of whether a production process is capable of producing items satisfying preset factory quality requirements, have received substantial research attention. Analysis of the process capability indices allows a production department to trace and improve a poor process, enhancing quality and satisfying customer requirements. Process capability analysis can also serve as an important reference for making decisions on improving the global quality of all products.

Kane (1986) considered the two basic indices C_p and C_{pk} and investigated some properties of their estimators. These indices are defined as follows:

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$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma},$$

where USL denotes the upper specification limit, LSL represents the lower specification limit, m is the midpoint of the specification interval (LSL, USL), $d = (USL - LSL)/2$, μ denotes the process mean, and σ represents the process standard deviation. As noted by Boyles (1994), C_p and C_{pk} are both yield-based indices, independent of the target T, and may fail to account for process centering. For this reason, Chan *et al.* (1988) developed index C_{pm}, which considers the process centering. Since C_{pm} was not originally designed to provide an exact measure of the number of non-conforming items, C_{pm} includes the process departure $(\mu - T)^2$ in the denominator of the definition (rather than 6σ alone) to reflect the degree of process centering. This index is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where T denotes the target value.

Pearn *et al.* (1998) considered the accuracy index $C_a = 1 - |\mu - T|/d$, which measures the degree of process centering. Process centering is defined as the ability of the process to cluster around the target value T. Generally, process centering can be measured by the departure of process mean μ from the target value T, namely, as $|\mu - T|$. Given the condition that the C_{pm} value is no less than a given level c, then the bound on C_a can be calculated as $C_a > 1 - (1/3c)$. Thus, given $C_{pm} > c$, the bounds on $|\mu - T|$ can be calculated as:

$$T - \frac{d}{3c} < \mu < T + \frac{d}{3c}.$$

Recently, many widely used statistical packages and quality researchers addressed process capability by applying C_{pm} to cases of asymmetric specification tolerances (see Kushler and Hurley (1992), and Franklin and Wasserman (1992)). As noted by Kotz and Johnson (1993), when the process is capable ($C_{pm} \geq 1$), the relationship between C_{pm} and process yield is $\%Yield \geq 2\Phi(3C_{pm}) - 1$. For example, if the capability of the product is 1.0, then the total process yield is guaranteed to exceed 99.73 percent.

Conversely, a smaller C_{pm} value implies higher expected loss, lower process yield, and poor process capability. Consequently, index C_{pm} is suitable for nominal-the-best type processes (bilateral specifications). Other indices like C_{pu} and C_{pl} are used for unilateral specifications processes. These two indices can be defined as:

$$C_{pu} = \frac{USL - \mu}{3\sigma},$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma},$$

Cheng (1994/1995) points out that the parameters of production process are unknown. Therefore, the estimated value of index must be obtained by means of sample. Because of error of sampling, the estimated value used to judge whether or not two processes are equally capable are not reliable. Thus, Chou (1994) developed a statistical test procedure using estimators of C_p, C_{pu} and C_{pl} to enable practitioners to determine whether or not two processes are equally capable. For bilateral specification process, index C_p failed to measure process yield and process centering. As noted by Schneider *et al.* (1995/1996), in the selection of qualified suppliers and during the certification process of potential supplier's processes, the customer's primary concern is to assure that the supplier is capable of producing consistently all material close to target. Thus, index C_{pm} can be used to develop a similar procedure for practitioners to use in determining whether or not two processes are equally capable. Of course, decisions made using the novel statistical test procedure to select better suppliers (or to evaluate whether or not the before and after improvable process are equally capable) are more reliable than decisions not based on the process.

The remainder of this paper is organized as follows. Section 2 introduces the estimation and the probability function of the process capability index. Section 3 then describes the hypothesis test for comparing two C_{pm} indices and also gives testing procedures. Subsequently, section 4 presents an example of the application of the procedure. Section 5 then outlines some conclusions.

2. Estimation

The parameters of the manufacturing processes are unknown. Directly observing processes and distinguishing the most capable process by point estimates seems to be too subjective. Large errors from sample data can occur, so the estimate one actually has at hand may not be very reliable (see Montgomery, 1985; Chou, 1994). Given product samples provided by two suppliers, sample data can be used to select the supplier providing the better product quality. Let $X_{i1}, X_{i2}, \ldots, X_{in_i}$, $i = 1, 2$ be measures of two samples independently drawn from the normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. The natural estimator of the process capability:

$$\text{index } C_{pmi}\left(C_{pmi} = \frac{d}{3\sqrt{\sigma_i^2 + (\mu_i - T)^2}}\right) \text{ is}$$

$$\hat{C}_{pmi} = \frac{d}{3\sqrt{S_i^2 + (\bar{X}_i - T)^2}},$$

where $\bar{X}_i = (\Sigma_{j=1}^{n_i} X_{ij})/n_i$ and $S_i^2 = \Sigma_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2/n_i$ represent the sample mean and variance of process i, for $i = 1, 2$, which may be obtained from the stable processes (processes are in-control). Table I briefly summarizes the above.

As denoted by Boyles (1991), assuming normality, the quantity $v_i(C_{pmi}/\hat{C}_{pmi})^2$ is approximately distributed as a chi-square distribution with v_i degrees of freedom, denoted by $\chi^2(v_i)$, where:

$$v_i = \frac{n_i(1 + [(\mu_i - T)/\sigma_i]^2)^2}{1 + 2[(\mu_i - T)/\sigma_i]^2}.$$

Since the process parameters μ_i and σ_i are unknown, v_i is also unknown. But, v_i can be estimated by calculating the values $\hat{v}_i$ from the sample, where:

$$\hat{v}_i = \frac{n_i(1 + [(\bar{X}_i - T)/S_i]^2)^2}{1 + 2[(\bar{X}_i - T)/S_i]^2}.$$

3. Test for comparing two C_{pm} indices

The formula for C_{pm} is easy to understand and apply. However, since the process measurements μ and σ are often estimated from the sample data to calculate the index value, significant uncertainty may be introduced into capability assessments due to sampling errors. For nominal-the-best type processes, Chou (1994) developed a procedure using estimators of C_p to allow practitioners to determine whether or not two processes are equally capable, making it possible to select the supplier with the better quality product. Since index C_p fails to measure process yield and process centering, index C_{pm} is used to develop a similar procedure for use in determining whether two processes are equally capable. The new procedure can be used to evaluate whether the before and after improvable process are equally capable. This study tests the null hypothesis H_o that $C_{pm1} = C_{pm2}$ against the alternative hypothesis H_a that $C_{pm1} \neq C_{pm2}$, equivalent to testing:

Sample	Average	Variance	Estimator
$X_{11}, \ldots, X_{1n_1}$	$\bar{X}_1 = \frac{\sum_{j=1}^{n_1} X_{1j}}{n_1}$	$S_1^2 = \frac{\sum_{j=1}^{n_1}(X_{1j} - \bar{X}_1)^2}{n_1}$	$\hat{C}_{pm1} = \frac{d}{3\sqrt{S_1^2 + (\bar{X}_1 - T)^2}}$
$X_{21}, \ldots, X_{2n_2}$	$\bar{X}_2 = \frac{\sum_{j=1}^{n_2} X_{2j}}{n_2}$	$S_2^2 = \frac{\sum_{j=1}^{n_2}(X_{1j} - \bar{X}_2)^2}{n_2}$	$\hat{C}_{pm2} = \frac{d}{3\sqrt{S_2^2 + (\bar{X}_2 - T)^2}}$

Table I. Sample data for the two stable processes

H_o. $C_{pm1} = C_{pm2}$ (two processes are equal capability).

H_a. $C_{pm1} \neq C_{pm2}$ (two processes are unequal capability).

The test statistic is given by:

$$F = \left(\frac{\hat{C}_{pm1}}{\hat{C}_{pm2}}\right)^2.$$

Assuming that H_o is true ($C_{pm1} = C_{pm2}$), the test statistic can be rewritten as:

$$F = \left(\frac{C_{pm2}/\hat{C}_{pm2}}{C_{pm1}/\hat{C}_{pm1}}\right)^2.$$

Using $\hat{v}_i$ to estimate v_i, then $(C_{pmi}/\hat{C}_{pmi})$ is approximately distributed as $\chi^2(\hat{v}_i)/\hat{v}_i$, the test statistic F is approximately distributed as F-distribution with $\hat{v}_2$ and $\hat{v}_1$ degrees of freedom when $C_{pm1} = C_{pm2}$. For level α test, we reject H_o in favor H_a if:

$$F < F_{\alpha/2}(\hat{v}_2, \hat{v}_1) \text{ or } F > F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1),$$

where F_α $(\hat{v}_2, \hat{v}_1)$ denotes the lower αth percentile of the F-distribution with $\hat{v}_2$ and $\hat{v}_1$ degrees of freedom. To evaluate the effect of this approximation, the simulation process is repeated $n = 1,000$ times by the Monte-Carlo method. The simulated parameter combinations are given in Table II. For each combination of (μ_1,σ_1) and (μ_2,σ_2), two random samples of size $n_1 = n_2 = n$, $n = 10(10)90$ were randomly drawn from processes 1 and 2, a estimated probability of type I error was then constructed. The test statistic F is recorded if $F < F_{\alpha/2}(\hat{v}_2, \hat{v}_1)$ or $F > F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1)$.

The proportion of times that the 1,000 values of F was less than $F_{\alpha/2}$ $(\hat{v}_2, \hat{v}_1)$ or greater than $F_{1-\alpha/2}$ $(\hat{v}_2, \hat{v}_1)$ can be calculated. This estimated α value (that is, estimated probability of type I error,) could then be compared to the level $\alpha = 0.05$. All calculated results are run using SAS program language. Table III displays the estimated α value for every parameter combination versus each sample size. The coverage frequency for the confidence interval is binomially distributed with $n = 1,000$ and $p = 0.05$. Hence, the 99 percent confidence interval for the coverage proportion is:

Table II. The simulated parameter combinations given $C_{pm1} = C_{pm2} = 1$ as H_0 is true

(μ_1, σ_1)	(μ_2, σ_2)
(12,000, 166.67)	(12,000, 166.67)
(11,850, 72.65)	(11,950, 158.99)
(12,100, 133.33)	(12,160, 46.67)

$$0.05 \pm 2.575\sqrt{\frac{0.05 \times 0.95}{1000}} = 0.05 \pm 0.018,$$

and it is possible to be 99 percent confident that a "true 95 percent confidence interval" will have a proportion of coverage of between 0.032 and 0.068. All the estimated α values in Table III are included between 0.032 and 0.068, and this is to believe the approximation is effective and reliable.

In order to consider the sensitivity of the test procedure, the power function of the test can be also computed as follows:

$$\text{power}(C_{pm2}/C_{pm1}) = p\left\{\left(\frac{C_{pm2}/\hat{C}_{pm2}}{C_{pm1}/\hat{C}_{pm1}}\right)^2 < \left(\frac{C_{pm2}}{C_{pm1}}\right)^2 F_{\alpha/2}(\hat{v}_2, \hat{v}_1)\middle| C_{pm1} \neq C_{pm2}\right\}$$

$$+ p\left\{\left(\frac{C_{pm2}/\hat{C}_{pm2}}{C_{pm1}/\hat{C}_{pm1}}\right)^2 > \left(\frac{C_{pm2}}{C_{pm1}}\right)^2 F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1)\middle| C_{pm1} \neq C_{pm2}\right\}.$$

The complete testing procedure is summarized in step form as follows:

(1) *Step 1*. Determine the sample size n_i for each process and the α-risk (normally set to 0.05), thus revealing the chance of rejecting a true H_o.

(2) *Step 2*. Take a random sample from each process and calculate the sample mean $\bar{X}_i = (\Sigma_{j=1}^{n_i} X_{ij})/n_i$ and sample variance $S_i^2 = \Sigma_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2/n_i$ of process i, for $i = 1, 2$.

(3) *Step 3*. Calculate the values of $\hat{v}_i$, $\hat{C}_{pmi}$, ($i = 1, 2$) and the test statistic F, where:

$$\hat{v}_i = \frac{n_i(1 + [(\bar{X}_i - T)/S_i]^2)^2}{1 + 2[(\bar{X}_i - T)/S_i]^2},$$

(μ_1, σ_1)	(μ_2, σ_2)	Sample size ($n_1 = n_2 = n$) 10	20	30	40	50	60	70	80	90
(12,000, 166.67)	(12,000, 166.67)	0.061	0.043	0.046	0.040	0.040	0.044	0.049	0.046	0.047
	(11,950, 158.99)	0.049	0.042	0.047	0.044	0.043	0.047	0.047	0.045	0.049
	(12,160, 46.67)	0.058	0.047	0.050	0.043	0.053	0.051	0.044	0.043	0.055
(11,850, 72.65)	(12,000, 166.67)	0.043	0.037	0.051	0.052	0.041	0.057	0.054	0.038	0.048
	(11,950, 158.99)	0.056	0.057	0.061	0.052	0.054	0.052	0.053	0.042	0.047
	(12,160, 46.67)	0.064	0.062	0.061	0.059	0.055	0.052	0.052	0.052	0.044
(12,100, 133.33)	(12,000, 166.67)	0.062	0.050	0.053	0.050	0.051	0.046	0.051	0.057	0.051
	(11,950, 158.99)	0.056	0.053	0.050	0.048	0.052	0.051	0.044	0.055	0.050
	(12,160, 46.67)	0.058	0.057	0.055	0.056	0.052	0.053	0.050	0.054	0.049

Table III. The estimated α value for every parameter combination vs various sample sizes

$$\hat{C}_{pmi} = \frac{d}{3\sqrt{S_i^2 + (\bar{X}_i - T)^2}},$$

$$F = \left(\frac{\hat{C}_{pm1}}{\hat{C}_{pm2}}\right)^2.$$

(4) *Step 4*. Decision rule:
- If $F_{\alpha/2}(\hat{v}_2, \hat{v}_1) \le F \le F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1)$, then do not reject H_o and conclude $C_{pm1} = C_{pm2}$.
- If $F < F_{\alpha/2}(\hat{v}_2, \hat{v}_1)$, then reject H_o and conclude $C_{pm1} < C_{pm2}$.
- If $F > F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1)$, then reject H_o and conclude $C_{pm1} > C_{pm2}$.

4. Application of the novel procedure to a color STN display process

To illustrate how the testing procedure can be applied to actual factory data, the following case study involving a color STN (Super Twist Nematic) displays product was taken from a manufacturing industry located in the Taichung economic processing zone, Taiwan. Color STN displays are created by adding color filters to traditional monochrome STN displays. Figure 1 displays the structure of color STN displays (sited www.wintek.com.tw). In color STN

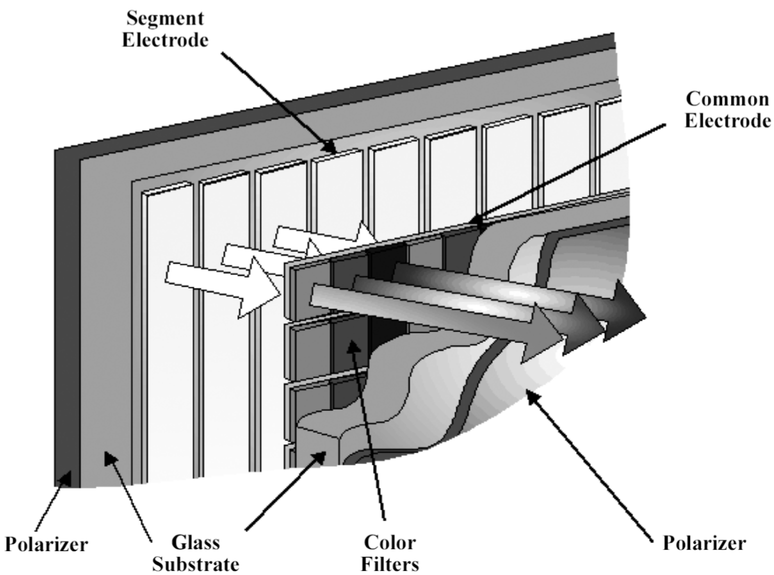

Figure 1. The structure of color STN displays

displays, each pixel is divided into R, G and B sub-pixels. Controlling the light through the color filter allows different colors to be produced through combinations of the primary colors. Figure 2 illustrates the manufacturing process for color filters.

Following the completion of the post baking process, the membrane thickness of each pixel is measured, and is an important quality characteristic focused on in this study. The specification limits are $12,000 \pm 500A^0$ ($1A^0 = 10^{-7}$mm), that is, the upper and the lower specification limits are set to $USL = 12,500$, $LSL = 11,500$. Meanwhile, the target value is set to $T = 12,000$. If the thickness of membrane does not fall within the tolerance (*LSL*, *USL*), color STN displays will suffer chromatic aberration. To compare product quality before and after improvement, 60 random samples from both before and after improvement are taken by a process engineer, and the sample data are listed in Table IV. Figure 3 and Figure 4 display the normal probability plots for the two collected data. We perform Kolmogorov-Smirnov test for normality check, obtaining $D_1 = 0.063861$ with p-value > 0.15 (from before improvement), and $D_2 = 0.102723$ with p-value > 0.1148. Since two p-value are sufficient large, we conclude that the before and after improvement are both normal. This study tests the null hypothesis H_o that $C_{pm1} = C_{pm2}$ against the alternative hypothesis H_a that $C_{pm1} \neq C_{pm2}$, where C_{pm1} and C_{pm2} represent the process capability indices before and after improvement. The test is equivalent to testing:

H_o. $C_{pm1} = C_{pm2}$ (before and after process are equally capable).

H_a. $C_{pm1} \neq C_{pm2}$ (before and after process are not equally capable).

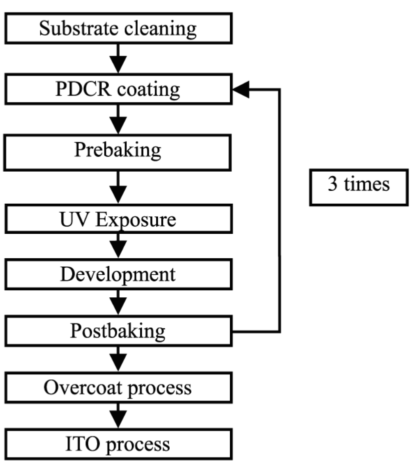

Figure 2.
The manufacturing process of color filters

Table IV.
The collected sample data with 60 observations

After improvement				Before improvement			
12,016	12,014	11,987	11,990	12,093	12,130	12,105	12,088
12,003	12,019	11,983	12,005	12,115	12,086	12,099	12,084
11,980	11,985	11,992	12,006	12,114	12,125	12,102	12,092
11,982	11,979	12,014	12,018	12,120	12,062	12,087	12,092
12,007	11,999	11,979	11,986	12,095	12,078	12,133	12,090
12,014	12,013	11,974	12,017	12,114	12,114	12,114	12,080
11,995	11,986	12,001	11,968	12,097	12,105	12,094	12,086
12,012	12,013	11,982	12,016	12,108	12,138	12,103	12,094
11,985	11,999	12,008	11,987	12,120	12,090	12,083	12,068
11,983	12,000	11,991	11,994	12,106	12,056	12,108	12,107
12,006	12,005	12,000	12,010	12,100	12,133	12,094	12,067
11,980	12,000	11,990	12,005	12,108	12,114	12,101	12,082
11,995	12,003	11,996	12,006	12,094	12,076	12,099	12,107
11,996	12,010	11,975	11,987	12,109	12,101	12,093	12,038
12,021	12,006	11,975	12,014	12,086	12,084	12,128	12,122

Note: Unit: A^0

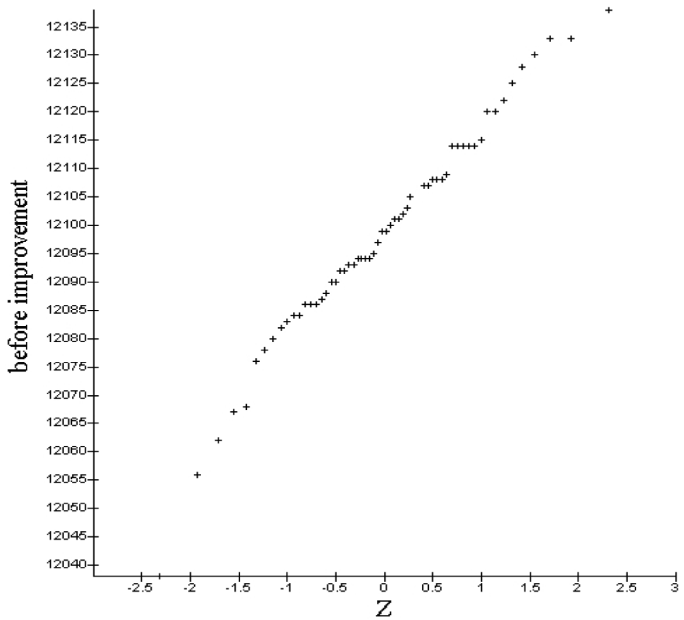

Figure 3.
The normal probability plot for the collected data from before improvement

The testing procedure is as follows:

(1) *Step 1*. Determine the sample size $n_1 = n_2 = 60$ before and after improvable process, with the significance level set at 0.05.

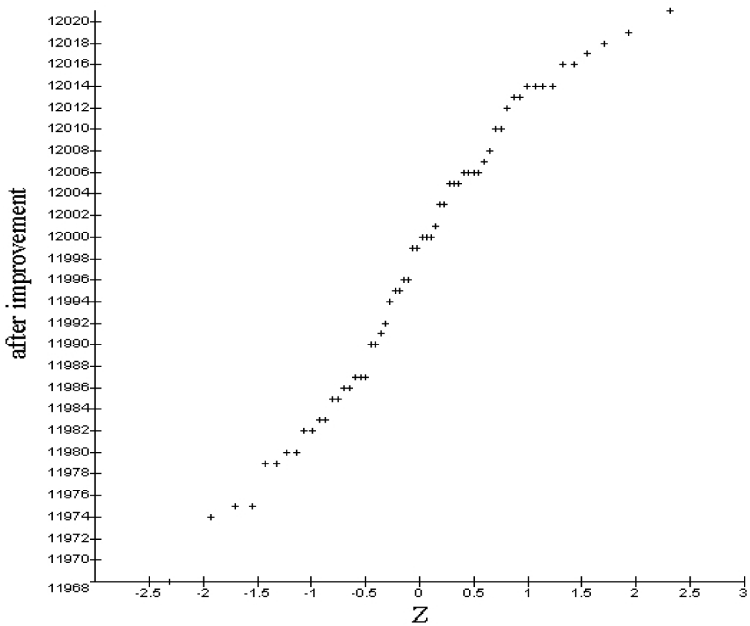

Figure 4.
The normal probability plot for the collected data from after improvement

(2) *Step 2*. From Table I, calculate the values of $\bar{X}_1$, $\bar{X}_2$, S_1^2, and S_2^2, where:

$$\bar{X}_1 = \frac{\sum_{j=1}^{n_1} X_{1j}}{n_1} = 12,098.52,$$

$$\bar{X}_2 = \frac{\sum_{j=1}^{n_2} X_{2j}}{n_2} = 11,997.70,$$

$$S_1^2 = \frac{\sum_{j=1}^{n_1} (X_{1j} - \bar{X}_1)^2}{n_1} = 369.82,$$

$$S_2^2 = \frac{\sum_{j=1}^{n_2} (X_{1j} - \bar{X}_2)^2}{n_2} = 184.98.$$

(3) *Step 3*. Calculate the values of $\hat{v}_1$, $\hat{v}_2$, $\hat{C}_{pm1}$, $\hat{C}_{pm2}$, and the test statistic F, where:

$$\hat{v}_1 = \frac{n_1(1 + [(\bar{X}_1 - T)/S_1]^2)^2}{1 + 2[(\bar{X}_1 - T)/S_1]^2} = 845.95,$$

$$\hat{v}_2 = \frac{n_2(1 + [(\bar{X}_2 - T)/S_2]^2)^2}{1 + 2[(\bar{X}_2 - T)/S_2]^2} = 60.05,$$

$$\hat{C}_{pm1} = \frac{d}{3\sqrt{S_1^2 + (\bar{X}_1 - T)^2}} = 1.66093,$$

$$\hat{C}_{pm2} = \frac{d}{3\sqrt{S_2^2 + (\bar{X}_2 - T)^2}} = 12.1809,$$

$$F = \left(\frac{\hat{C}_{pm1}}{\hat{C}_{pm2}}\right)^2 = 0.018590.$$

(4) *Step 4*. Calculate $F_{\alpha/2}(\hat{v}_2, \hat{v}_1) = F_{0.025}(60.05, 845.95) = 0.66799$, $F_{1-\alpha/2}(\hat{v}_2, \hat{v}_1) = F_{0.975}(60.05, 845.95) = 1.40877$.

(5) *Step 5*. The power curve of the test is depicted in Figure 5 under the ratio value of $C_{pm2}/C_{pm1} = 0.4$ to 1.6.

Because $F = 0.018590 < 0.66799$, we conclude that the improved process is more capable than the unimproved process.

5. Conclusions

Chou (1994) developed a procedure using estimators of C_p, C_{pu} and C_{pl} to determine whether or not two processes are equally capable. For bilateral specification processes, index C_p failed to measure process yield and process

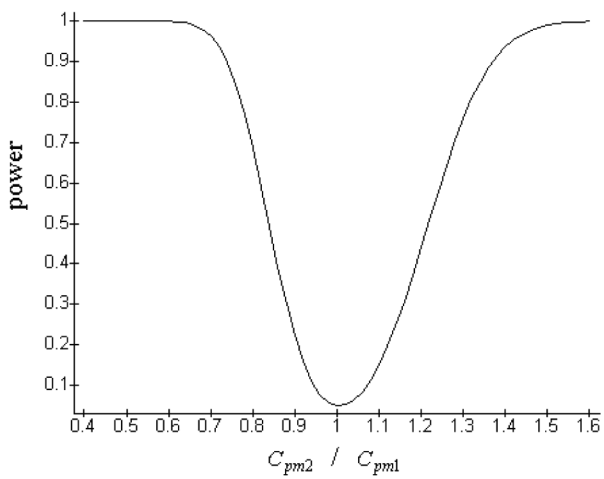

Figure 5.
Power curve of testing

centering. Consequently, this study uses the index C_{pm} to develop a similar procedure for determining whether or not two processes are equally capable. The new procedure can be used to evaluate whether or not the processes equally capable before and after improvement. Decisions made by using the novel procedure to select better suppliers (or to evaluate the capability of processes before and after improvement) are naturally more reliable than decisions made without the help of the procedure. This study introduces the estimation and probability function of the process capability index, as well as the hypothesis test for comparing the two C_{pm} indices and testing procedures. Finally, an example of the application of the novel procedure is provided, using data from a manufacturing industry located in the Taichung economic processing zone, Taiwan.

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